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Dark Matter and Large Scale Structure

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Abstract. A review of the study of dark matter and large scale structure of the Universe at Tartu Observatory is given. Tartu astronomers have participated in this development, starting from Ernst Öpik and Grigori Kuzmin, and continuing with the present generation of astronomers. Our goal was to understand better the structure, origin and evolution of the Universe.

1. Prologue

Once I happened to read the Thomas Kuhn (1970) book *The Structure of Scientific Revolutions*. The presence of dark matter around galaxies had just been reported, and the fight between supporters and opponents of the dark matter concept was at its peak. Then I realised that the dark matter story seems to be a good example of a scientific revolution. Ten years later, in concluding remarks of the IAU Symposium on “Dark Matter in the Universe”, Scott Tremaine also pointed to the development of the dark matter concept as a classical example of a scientific revolution (Tremaine 1987, see also Binney & Tremaine 1987).

Tartu astronomers have participated in the dark matter study for a long time, starting from Ernst Öpik – the founder of the contemporary astronomy school in Estonia; and followed by Grigori Kuzmin, the most talented student of Ernst Öpik. The present generation of astronomers has continued the study of dark matter. Unexpectedly, this work has lead us to study the distribution of galaxies and clusters on large scales and led to the discovery of voids and filaments.

In the following I shall give a review of the study of dark matter and large scale structure of the Universe, as seen by an Estonian astronomer. This story is biased, and is to be complemented by similar stories of the development of the dark matter concept elsewhere. However, I hope that our story is of interest to the cosmology community, as some aspects of it are not well documented so far.

2. Local Dark Matter

2.1. Ernst Öpik

An astrophysical approach in astronomy was introduced at Tartu Observatory with the work by Ernst Öpik. He started his studies being a student of Moscow University. One of the first questions he was curious about was the possible

presence of invisible matter in our Galaxy. He developed a method to determine the density of matter near the Galactic plane using vertical oscillations of stars. He concluded that there is no evidence for large amounts of invisible matter near the Galactic plane (Öpik 1915).



Figure 1. Ernst Öpik with his wife Alice and author in Brighton during the IAU General Assembly 1970.

A hot topic was the discussion on the nature of spiral nebulae: Are they gaseous objects within the Milky Way or distant worlds similar in structure to our Galaxy? Öpik was also interested in this problem. In 1918 he delivered a talk at the Meeting of the Moscow Society of Amateur Astronomers, devoted to his study of the structure of the Andromeda Nebula, M31. Shortly before, the first relative velocity measurements of the central bulge of M31 had been published, and Öpik had developed a method for estimating distances to spiral nebulae from relative velocities within them. He used Newton's law of gravity, and the apparent luminosity of the central part of M31 derived from photometric observations. Further he assumed that the mass-to-luminosity ratio in M31 is identical to the ratio in our Galaxy, and developed a mass distribution model of M31 to estimate a parameter in the formula for relative velocities. He accepted for the mass-to-luminosity ratio in the Solar vicinity a value 2.63 in Solar units (Kapteyn 1904, van Rhijn 1920), and Öpik's (1915) own estimate of the mass density of the matter in the Solar neighbourhood. From these data he got for the distance of the Andromeda Nebula 440 kpc (Öpik 1922). With this work he solved the problem of the nature of spiral nebulae, and developed mass

distribution models of galaxies – a prerequisite for further work which culminated 50 years later with the discovery of dark halos of galaxies.

2.2. Grigori Kuzmin

The work on galactic mass modelling was continued by Grigori Kuzmin. In the 1940s he developed a new method for galactic mass modeling. He applied his theory first to the Andromeda galaxy using recently published rotation data by Babcock (1939). Due to war time he published his results only in Estonian in the yearbook of the Tartu Observatory (Kuzmin 1943).

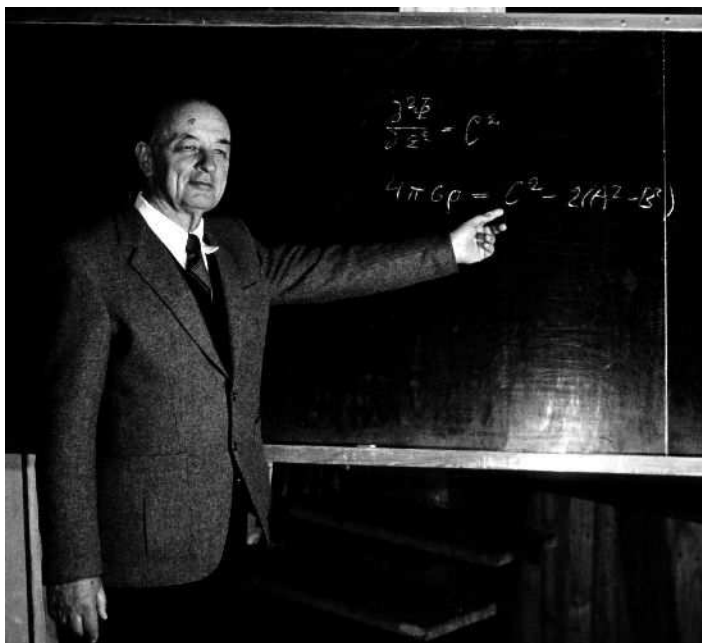


Figure 2. Grigori Kuzmin explaining his formula to calculate the dynamical density of matter in the Galaxy, late 1970s.

Next Kuzmin turned his attention to our own Galaxy. Here the central problem was the density of matter in the Solar vicinity. The mass density is given by the Poisson equation. Kuzmin expressed the derivative of the gravitational potential in the Galactic plane through Oort constants, A and B , and the derivative of the gravitational potential in the vertical direction, $\partial\Phi/\partial z$, through a constant C , which characterises the vertical gravitational acceleration; it has the same dimension as the A and B , and is called the Kuzmin constant. This derivative can be expressed through the ratio of dispersions of velocities and coordinates in vertical direction, $C = \sigma_z/\zeta_z$ (Kuzmin 1952a, 1955). Finally the Poisson equation takes the form:

$$4\pi G\rho = C^2 - 2(A^2 - B^2). \quad (1)$$

Kuzmin (1952a, 1955) used data on the distribution of A and gK stars and analysed results obtained in earlier studies by Oort (1932) and others. He got a

weighted mean value $C = 68 \text{ km s}^{-1} \text{ kpc}^{-1}$, which leads to the density estimate $\rho = 0.08 M_{\text{sun}} \text{ pc}^{-3}$, in good agreement with direct density estimates of all known stellar populations (including estimates for the mass in invisible low-mass stars and white dwarfs). Two students of Kuzmin made independent analyses, using different methods and initial data (Elsalu 1959, Jõeveer 1972, 1974b) and confirmed Kuzmin results. Thus we came to the conclusion that *there is no evidence for the presence of dark matter in the disk of the Galaxy*.

In parallel to the analysis of the spatial density Kuzmin developed a detailed model of the Galaxy (Kuzmin 1952b, 1953, 1956a, b). Here, for the first time, non-homogeneous ellipsoids were used instead of sums of ellipsoids of constant density; also the third integral of motion of stars was applied, and principles of calculation of 3-dimensional stellar orbits were presented. The mass model of the Galaxy by Schmidt (1956) still uses sums of ellipsoids of constant density; also a higher value of the local density was accepted. The local density problem was studied again by Hill (1960) and Oort (1960); both obtained considerably higher local densities of the matter, and argued that there exist large amounts of dark matter in the Galactic disk. The discrepancy between Tartu and Dutch results for the presence or absence of the local dark matter in the disk of the Galaxy was not solved until recently. Modern data have confirmed the results by Kuzmin and his collaborators (Gilmore, Wyse & Kuijken 1989).

3. Global Dark Matter

3.1. Galactic Models

Problems of the structure and evolution of stars and stellar systems were a central issue at Tartu Observatory. Öpik (1938) developed the modern theory of stellar evolution based on the burning of hydrogen in stellar cores, which leads to the formation of red giant stars after the main sequence stage. Rootsmäe (1961) applied these ideas to kinematics of stars to find the sequence of formation of different stellar populations; similar ideas were developed independently by Eggen, Lynden-Bell and Sandage (1962). Kuzmin developed the major principles of galactic modeling. Thus, in the 1950s we had a concept of the structure and evolution of stellar populations in galaxies which is rather close to the presently accepted picture. It was quite natural to apply this knowledge to construct more detailed models of galaxies, including explicitly main stellar populations.

My first experience of galaxy modeling was in 1952 when I made calculations for the Kuzmin new model of the Galaxy. At this time a popular model was presented by a leading Soviet astronomer Parenago (1950). In the Parenago model a circular velocity law was accepted which has a rather fast decrease on the periphery and led to negative spatial densities. I suggested an improvement of the model: we applied the velocity law only until the galactocentric distance where the density becomes zero; and we assumed zero density outside this limit. At larger distances the circular velocity was calculated from the density law.

I made my PhD thesis on kinematics of stars and returned to galactic modeling again in the 1960s. The modeling proceeded step by step. Detailed local structure is known only for our own Galaxy, and global information on stellar populations is better known for external galaxies. Thus it was natural to study our Galaxy and our next neighbour, the Andromeda galaxy, in parallel. The

first step was to find a balanced system of Galactic constants, using all available data. This system was presented in the Commission 33 Meeting of the 12th General Assembly of IAU (Einasto and Kutuzov 1964); it can be considered as the first version of the presently accepted IAU system of Galactic constants. A composite model of the Galaxy based on this system of constants was calculated (Einasto 1965). Then I realised that there was a need for a more detailed method for the construction of composite models of galaxies. This goal was realised in a series of papers in Tartu Publications, and a summary was published in a journal (Einasto 1969a). The main principles of model construction were: galaxies can be considered as sums of physically homogeneous populations; physical properties of populations (mass-to-luminosity ratio, colour) should be in agreement with models of physical evolution of stellar populations; the density of a population can be expressed as ellipsoids of constant flatness and rotational symmetry; densities of populations are non-negative and finite; moments of densities which define the total mass and effective radius of the galaxy are finite. It was found that in a good approximation densities of all stellar populations can be expressed by a generalised exponential law: $\rho(a) = \rho(0) \exp[-(a/a_c)^{1/N}]$, where $\rho(0) = hM/(4\pi\epsilon a_0^3)$ is the central density, $a = \sqrt{R^2 + z^2/\epsilon^2}$ is the distance along the major axis, ϵ is the axial ratio of the equidensity ellipsoid, $a_c = ka_0$ is the core radius (a_0 is the harmonic mean radius), h and k are normalizing parameters, depending on the structural parameter N , which allows to vary the density behaviour with a . The definition of normalizing parameters and their calculation was described by Einasto (1970b). The cases $N = 1$ and $N = 4$ correspond to conventional exponential and de Vaucouleurs models, respectively.

The method was applied for the Andromeda galaxy (Einasto 1969b, 1970a, Einasto & Rummel 1970a), and for our Galaxy (Einasto 1970b). In the case of the Andromeda galaxy we encountered two problems. The first problem concerns rotation data and physical properties near the centre of the galaxy. If we accepted rotational velocities, based mostly on radio observations (Roberts 1966), then the mass-to-luminosity ratio, M/L , of central stellar populations became very low, of the order of 1 in Solar units. On the other hand, spectral data (Spinrad 1966) suggested a much higher value, $M/L \approx 17$. To solve this discrepancy, we analysed the velocity field obtained from radio observations, and demonstrated that low rotational velocities in central regions are due to low spatial resolution of the radio beam (Einasto & Rummel 1970b,c). The corrected velocity field was in agreement with a higher value of M/L in central regions of M31. As a further test the rate of star formation as function of the density of stellar populations was studied in M31, and the results confirmed the Schmidt (1959) law: the star formation rate is proportional to the density squared (Einasto 1972).

Additionally, a model of physical evolution of stellar populations was developed (Einasto 1971). The model was similar to the model by Tinsley (1968), but some aspects were developed in more detail. The model used as input data the evolutionary tracks of stars of various composition (metallicity) and age; the star formation rate from Salpeter (1955) as a function of stellar mass was used, as well as a low-mass limit of star formation of $M_0 = 0.03 M_{sun}$. The model yielded a continuous sequence of population parameters as a function of age (colour, spectral energy distribution, M/L). The results of modeling

stellar populations were compared with direct dynamical data for central regions of galaxies (velocity dispersions) by Einasto & Kaasik (1973). These data supported relatively high values ($M/L \approx 10 - 30$) for old metal-rich stellar populations near centres of galaxies; moderate values ($M/L \approx 3 - 10$) for discs and bulges; and low values ($M/L \approx 1 - 3$) for metal-poor halo-type populations. Modern data yield slightly lower values, due to more accurate measurements of velocity dispersions in central regions of galaxies.

The second problem encountered in the modeling of M31 was the rotation and density distribution on the periphery. If rotation data were taken at face value, then it was impossible to represent the rotational velocity with the sum of known stellar populations. The local value of M/L increases towards the periphery of M31 very rapidly if the mass distribution is calculated directly from rotation velocity. All known old metal-poor halo-type stellar populations have a low $M/L \approx 1$; in contrast on the basis of rotation data we got $M/L > 1000$ on the periphery of the galaxy near the last point with measured rotational velocity.

There were two possibilities to solve this problem: to accept the presence of a new population with very uncommon properties, or to assume that on the periphery of galaxies there exist non-circular motions. We found that the first alternative has several serious difficulties. If the hypothetical population is of stellar origin, it must be formed much earlier than known populations, because all known stellar populations form a continuous sequence of kinematical and physical properties (Rootsmäe 1961, Einasto 1974a), and there is no place where to put this new population into this sequence. Secondly, the star formation rate is proportional to the square of the local density (Schmidt 1959, Einasto 1972), thus stars of this population should have been formed during the contraction phase of the formation of the population near its central more dense regions, and later expanded to the present distance. The only source of energy for expansion is the contraction of other stellar populations. The estimated total mass of the new population exceeded the summed mass of all previously known populations. Estimates of the energy needed for the expansion demonstrated that the mass of the new population is so large that even the contraction of all other stellar populations to zero radius would not be sufficient to expand the new population to its present size. And, finally, it is known that the star formation is not an efficient process (usually in a contracting gas cloud only about 1 % of the mass is converted to stars); thus we have a problem how to convert, in an early stage of the evolution of the Universe, a high fraction of primordial gas to this population of stars. Taking into account all these difficulties we accepted the second alternative – the presence on non-circular motions (Einasto 1969b), similar to many other astronomers (see Materne & Tammann 1976). As we soon realised, this was a wrong decision.

3.2. Galactic Coronae

In spring 1972 I was asked to give an invited review on Galactic models at the First European Astronomy Meeting in Athens. At this time population models of galaxies had been calculated already for 5 galaxies of the Local Group and the giant elliptical galaxy M87 in the Virgo cluster. More and more data accumulated on rotation velocities of galaxies. New data suggested the presence of almost flat rotation curves on the periphery of galaxies, thus it was increas-

ingly difficult to accept the previous concept of large non-circular motions. On the other hand, recently finished calculations of the physical evolution of stellar populations confirmed our previous view that it is extremely difficult to accept a stellar origin of the hypothetical population. In summer 1972 I discussed the problem with my collaborator Enn Saar. He suggested to abandon the idea that only stellar populations exist in galaxies, to assume that there is a population of unknown nature and origin and to look which properties it should have using available data on known stellar populations. Here we can say with the words of Sherlock Holmes *“When you have eliminated the impossible, whatever remains, however improbable, must be the truth”* (cited by Binney & Tremaine 1987). Quickly a second set of models for galaxies was calculated, and parameters for the new dark population were found (for the distribution of mass-to-luminosity ratio in galaxies see Figure 3). My talk in Athens was on September 8, 1972. The main results were (Einasto 1974b): (1) There are two dark matter problems: the local and the global one; (2) The local dark matter, if it exists, must be of stellar origin, as it is strongly concentrated to the Galactic plane; (3) The global dark matter is of non-stellar origin; it has very low concentration to the plane and centre of the galaxy; its dynamical and physical properties are different from all previously known stellar populations. To avoid confusion with the conventional halo population I suggested to call the new population “corona”; (4) Available data are insufficient to determine outer radii and masses of coronae. Preliminary estimates indicated that in some galaxies the mass and radius of the corona may exceed considerably the mass and radius of stellar populations.

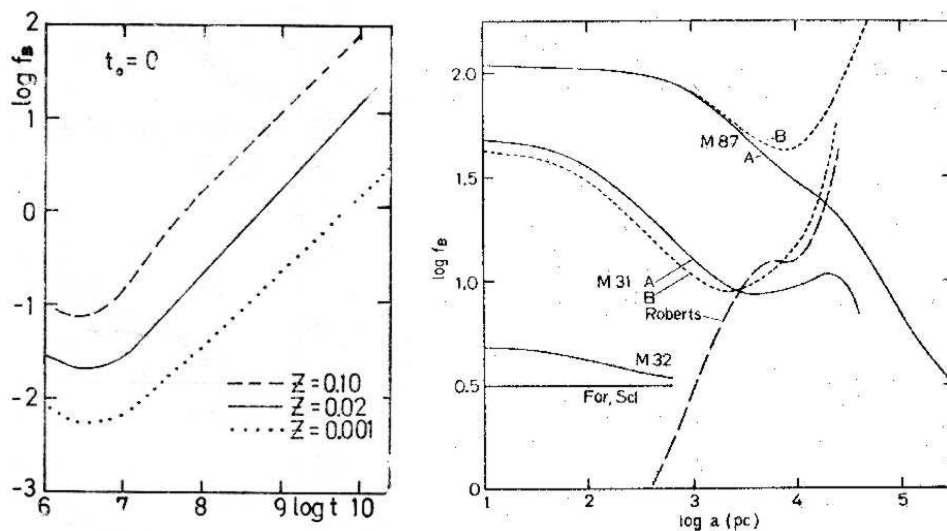


Figure 3. Left: evolution of the mass-to-luminosity ratio f_B for stellar populations of different metallicity Z and instant star formation (Einasto 1971). Age t is given in years. Right: Distribution of mass-to-luminosity ratio, $f_B = M/L_B$, in galaxies of the Local Group and M87: models without (A) and with (B) dark corona (Einasto 1974b).

The Athens report did not give rise to special excitement. The main reason for this lukewarm reception was probably the absence of a solid proof for the existence of the corona, of its main parameters (mass and radius), and of its nature. Thus I continued the search for further evidence. Soon I noticed that the problem of galactic coronae is the same as discussed already long time ago in clusters and groups of galaxies, starting from the pioneering work by Zwicky (1933) and Kahn and Woltjer (1959). The problem was discussed in detail at the Santa Barbara Conference on the Instability of Systems of Galaxies (Neyman, Page & Scott 1961). I fully agreed with arguments by van den Bergh (1961) that clusters of galaxies are old systems, thus they must be stabilised by large masses. A similar problem exists in double elliptical galaxies. The mean mass-to-luminosity ratio of double elliptical galaxies $M/L \approx 66$ (Page 1952, 1960), whereas, according to our recent population models, stellar populations had $1 \leq M/L \leq 30$. In particular, for bulges of galaxies our composite galactic model gave $M/L \approx 3 - 10$ in good agreement with models of physical evolution of stellar populations (see Figure 3). Low estimates for M/L in bulges were confirmed by Faber et al. (1977) who measured rotation velocities near the edge of the bulge of the Sombrero galaxy and found $M/L \approx 3$. Thus it was evident that elliptical galaxies should have massive coronae.

Reading these papers on the mass discrepancy in clusters and elliptical galaxies I realised how it is possible to check the presence of dark coronae around galaxies. If coronae are large enough, then in pairs of galaxies the companion galaxy can be considered as a test particle to measure the gravitational attraction of the main galaxy. Mean relative velocities, calculated for different distances from the main galaxy, can be used instead of rotation velocities to find the mass distribution of giant galaxies. Quickly I collected data for pairs of galaxies, and on January 11, 1974 I had the results: radii and masses of galactic coronae exceeded radii and masses of parent galaxies by an order of magnitude! Together with A. Kaasik and E. Saar we calculated new models of galaxies including dark coronae.

In those years Soviet astronomers had the tradition to gather in Caucasus Winter Schools. Results of galactic mass modeling were reported in a Winter School in 1972. That year the School was held near the Elbrus mountain in a winter resort. I had my report on the masses of galaxies on January 29, 1974. My message was: *since the data suggest that all giant galaxies have massive coronae, dark matter must be the dominating component in the whole universe.* In the Winter School prominent Soviet astrophysicists like Zeldovich, Shklovsky, Novikov and others participated. After the talk the atmosphere was as if a bomb had exploded. Everybody realised that, if true, this is a discovery of principal importance. Two questions dominated: What is the physical nature of the dark matter? and What is its role in the evolution of the Universe?

We had to hurry, since large masses of halos were already discussed by Ostriker and Peebles (1973). Preliminary results of our analysis were published in February 1974 by Einasto et al. (1974b) in *Astron. Tsirk.* But Zeldovich insisted that this is not enough: “Major results must be published in major journals”. Thus a more detailed report was sent to *Nature* (Einasto, Kaasik & Saar 1974) and, for the first time, a preprint was made and sent to all observatories. Soon we realised that it was just in time: Ostriker, Peebles and Yahil

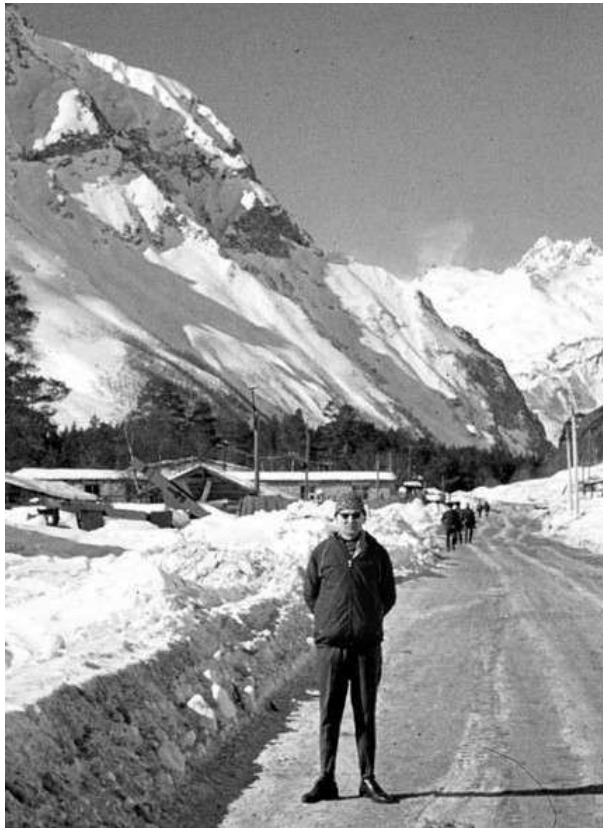


Figure 4. Author in the Caucasus winter school.

(1974) got similar results using similar arguments; their paper was published several months after our *Nature* paper and has a reference to our preprint. Soon the first reaction to our results appeared: Burbidge (1975) formulated difficulties of the dark corona concept. The main problem is in the statistical character of the dynamical determination of the mass of double galaxies. If companion galaxies used in mass determination are not real physical companions but random interlopers, then the mean velocity dispersion reflects random velocities of field galaxies and no conclusions on the mass distribution around giant galaxies can be made. The latter three publications initiated the dark matter boom.

Difficulties connected with the statistical character of our arguments were discussed already in the Winter School, thus we started immediately a study of properties of companion galaxies to find evidence for some other regularity in the satellite system which surrounds giant galaxies. Soon we discovered that companion galaxies are segregated morphologically: elliptical (non-gaseous) companions lie close to the primary galaxy whereas spiral and irregular (gaseous) companions of the same luminosity have larger distances from the primary galaxy; the distance of the segregation line from the primary galaxy depends on the luminosity of the primary galaxy (Einasto et al. 1974a, see Fig-

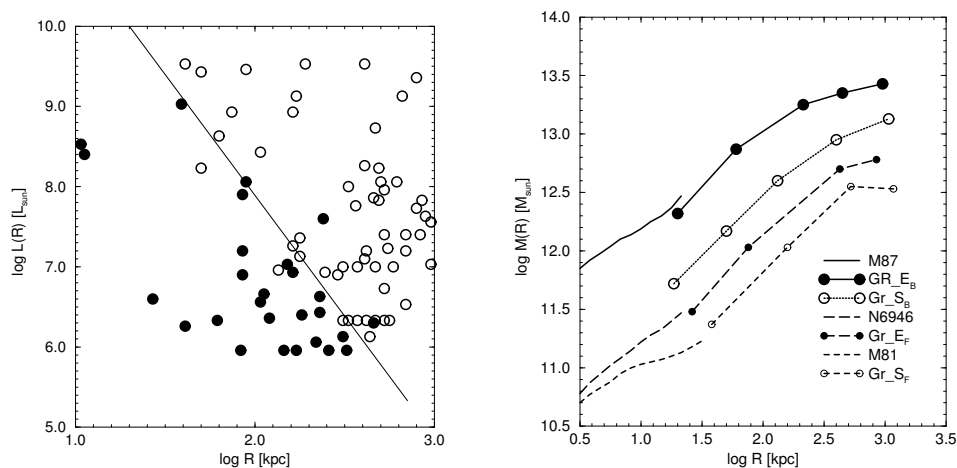


Figure 5. Left: Distribution of luminosity of companion galaxies of different morphology vs. distance from the central galaxy; spiral and irregular companions are marked with open circles, elliptical companions with filled circles (Einasto et al. 1974). Right: Distribution of internal mass in the giant elliptical galaxy M87, giant spiral galaxy NGC6946, and medium luminous spiral galaxy M81, compared with mass distribution in groups of galaxies derived from relative motions of companions of giant and medium bright elliptical and spiral galaxies (Einasto et al. 1975).

ure 5). This result shows, first of all, that companions are real members of these systems – random by-flyers cannot have such properties. Second, this result demonstrated that diffuse matter can have a certain role in the evolution of galaxy systems. The role of diffuse matter in galactic coronae was discussed in detail by Chernin, Einasto & Saar (1976). Morphological properties of companion galaxies can be explained, if we assume that, at least part of the corona is gaseous. On the other hand, Komberg & Novikov (1975) demonstrated that coronae cannot be fully gaseous. Thus the nature of coronae remained unclear. Also we found that dynamical and morphological properties of primary galaxies are well correlated with properties of their companions (Einasto et al. 1976c, see Figure 5). A further evidence of the large mass of the corona of our Galaxy came from the study of the dynamics of the Magellanic Stream (Einasto et al. 1976a). The stellar component of Galactic coronae was discussed by Jõeveer (1974a) and Jaaniste & Saar (1975). The publication story of the last paper is interesting. First authors submitted the paper to *Astrophysics and Space Science*, but the editor S.B. Pikelner rejected the paper with justification: “you already have a paper on dark matter” (by Chernin, Einasto & Saar 1976). He could not imagine that in years to come thousands of papers will be written on this subject.

In January 1975 the first conference on dark matter was held in Tallinn, Estonia. The rumor on dark matter had spread around the astronomical community and, in contrast to conventional local astronomy conferences, leading

Soviet astronomers and physicists attended. Main topics were new data for the evidence of the dark matter, and its possible nature. Here also statistical arguments against the dark matter concept were presented by Fessenko. The next dark matter discussion was in July 1975 during the Third European Astronomical Meeting in Tbilisi, Georgia, where a full session was devoted to the dark matter problem. The principal discussion was between our group (Einasto et al. 1976d) and Materne & Tammann (1976). Historically, this was the first well documented discussion between supporters and opponents of the galactic dark matter concept. Tammann ignored our arguments, as well as arguments by Zwicky, Kahn and Woltjer. Using his own statistical data he concluded that systems of galaxies are stable with conventional masses. It was clear that by sole dispute it is not possible to solve the problem – new data were needed. These data were supplied by Vera Rubin and her collaborators in their analysis of rotation curves of galaxies (Rubin, Ford & Thonnard 1978, 1980, see also a review by Rubin 1987). These studies indicated that practically all spiral galaxies have flat rotation curves as expected in the presence of dark coronae (halos).

Dark matter problems were also discussed during the IAU General Assembly in Grenoble, 1976. Here arguments for the non-stellar nature of dark coronae were again presented (Einasto, Jõeveer & Kaasik 1976b). I remember that after my talk Ivan King quietly said from the audience *“Perhaps really there are two halos of galaxies”* (stellar halo and non-stellar corona). The concept of the dark matter was, however, generally accepted by theorists, only after the need for dark matter in the evolution of the Universe was clarified. Rees (1977) noticed that neutrinos can be considered as dark matter particles; and Chernin (1981) showed that, if dark matter is non-baryonic, then this helps to explain the paradox of small temperature fluctuations of cosmic microwave background radiation. Density perturbations of non-baryonic dark matter start growing already during the radiation-dominated era whereas the growth of baryonic matter is damped by radiation. If non-baryonic dark matter dominates dynamically, the total density perturbation can have an amplitude of the order 10^{-3} at the recombination epoch, which is needed for the formation of the observed structure of the Universe. This problem was discussed in a conference in Tallinn in April 1981. Here all prominent Soviet cosmologists and particle physicists participated (this conference was probably the birth of the astro-particle physics). The central problem was the nature of the dark matter. In the conference banquet Zeldovich held an enthusiastic speech: *“Observers work hard in sleepless nights to collect data; theorist interpret observations, are often in error, correct their errors and try again; and there are only very rare moments of clarification. Today it is one of such rare moments when we have a holy feeling of understanding secrets of the Nature.”* Non-baryonic dark matter is needed to start structure formation early enough. This example illustrates well the attitude of theorists to new observational discoveries – the Eddington’s test: *“No experimental result should be believed until confirmed by theory”* (cited after Turner 1999). Now, finally, the presence of dark matter was accepted by leading theorists.

This was, however, not the end of the story. Soon it was realized that neutrino-dominated or hot dark matter generates almost no fine structure of the Universe. But this is already part of the next story.

4. Large-Scale Structure of the Universe

4.1. Zeldovich Question

After my talk in the Caucasus Winter School in 1972 Zeldovich turned to me and offered collaboration in the study of the universe. He was developing a theory of the formation of galaxies (the pancake theory); an alternative whirl theory was suggested by Ozernoy, and a third theory of hierarchical clustering by Peebles. Zeldovich asked for our help in solving the question: Can we find some observational evidence which can be used to discriminate between these theories?



Figure 6. Yakov Zeldovich with his wife visiting Estonia, late 1970s.

So far we had no experience in observational cosmology, our work was directed to the understanding of the structure of galaxies. We had theoretical cosmologists in our group (Enn Saar and Jaak Jaaniste), but they also did not have experience in observational cosmology. Thus, initially we had no idea how we can help Zeldovich. But soon we remembered our previous experience in the study of galactic populations: kinematical and structural properties of populations hold the memory of their previous evolution and formation (Rootsmäe 1961, Eggen, Lynden-Bell & Sandage 1962). Random velocities of galaxies are of the order of several hundred km/s, thus during the whole lifetime of the Universe galaxies have moved from their place of origin only about $1 h^{-1}$ Mpc (we use in this paper the Hubble constant in units $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$). In other words – if there exist some regularities in the distribution of galaxies, these regularities must reflect the conditions in the Universe during the formation of

galaxies. Actually we had already some first results: the study of companion galaxies had shown that dwarf galaxies are located almost solely around giant galaxies and form together with giant galaxies systems of galaxies. In other words – the formation of galaxies occurs in larger units, not in isolation.

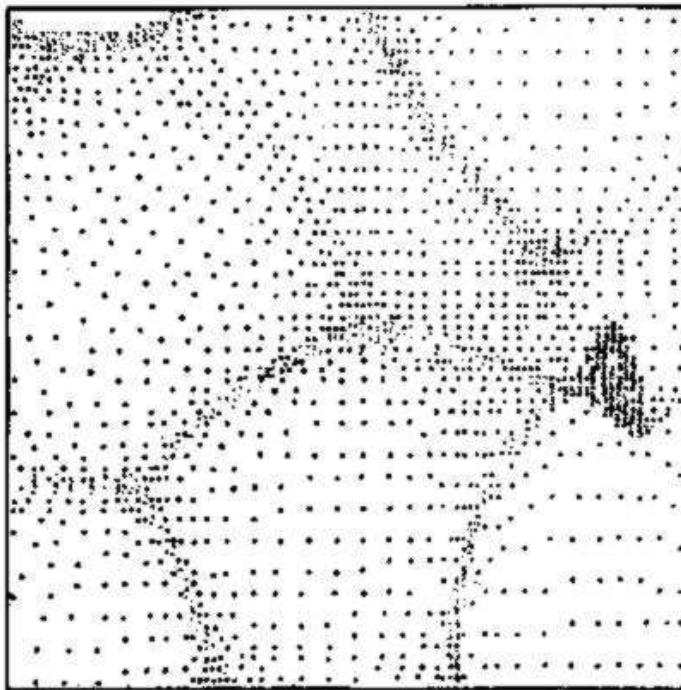


Figure 7. Distribution of particles in simulations (Doroshkevich, Shandarin and Novikov 1975).

4.2. Superclusters, Voids and Filaments

In our work to solve the Zeldovich question we had close collaboration with his team. In 1975 Doroshkevich, Shandarin and Novikov obtained first results of numerical simulations of the evolution of particles according to the theory of gravitational clustering developed by Zeldovich (1970). This was a 2-dimensional simulation with 32×32 particles; a figure with results of these simulations was put on the wall of the Saar and Jaaniste office (see Figure 7). In this picture a system of high- and low-density regions was seen: high-density regions form a cellular network which surrounds large under-dense regions. One of our challenges was to find out whether the real distribution of galaxies showed some similarity with the theoretical picture.

Now we had a leading idea how to solve the problem of galaxy formation: *We have to study the distribution of galaxies on larger scales.* Both our galactic astronomy and theoretical cosmology groups participated in this effort. One approach we used was the study of the distribution of nearby Zwicky clusters.

Many bright galaxies of nearby Zwicky clusters had at this time measured redshifts, so we hoped to determine the distribution of clusters and to find some regularities there. To see the distribution better we built in the office of Saar and Jaaniste a 3-dimensional model from plastic balls. Some regularity was evident: there were several clusters of Zwicky clusters – superclusters, one of them in the Perseus region. But too many clusters had no galaxies with measured redshifts, so it was difficult to get an overall picture.

A different approach was used by Mihkel Jõeveer. He used wedge-diagrams, invented just when we started our study. His trick was: he made a number of relatively thin wedge diagrams in sequence, and plotted in the same diagram galaxies, as well as groups and clusters of galaxies. In these diagrams a regularity was clearly seen: *isolated galaxies and galaxy systems populated identical regions, and the space between these regions was empty*. After this success the whole group continued the study using wedge-diagrams. Most attention was directed to the Perseus supercluster, well seen as a chain of Zwicky and Abell clusters. To our great surprise and joy slices with galaxies and clusters were quite similar to the predicted picture. We made wedge-diagrams for the full sky, in declination and right ascension (see Figure 8), and a very detailed analysis of the Perseus supercluster region, where the number of foreground galaxies was very small (see Figure 9).

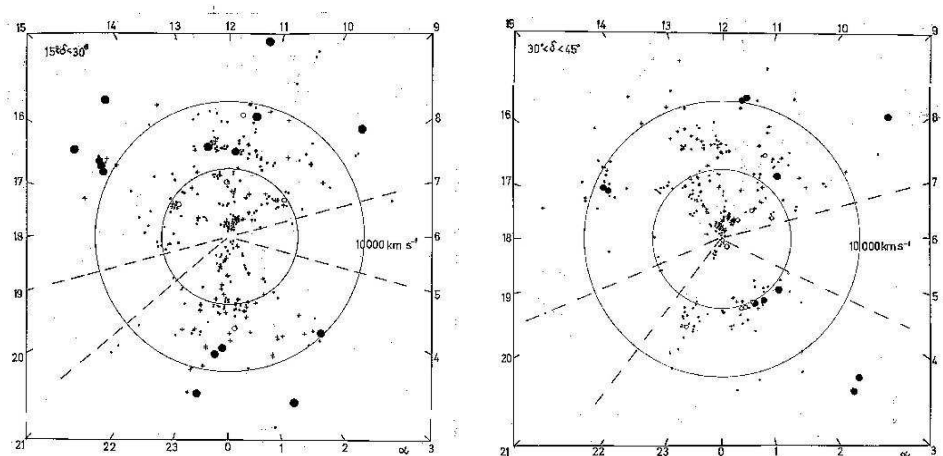


Figure 8. Wedge diagrams of galaxies (dots), radio galaxies (crosses), clusters (filled circles) and groups (open circles) in two declination zones (Jõeveer and Einasto 1978). On left panel we see the Local Supercluster near the center, the Coma and the Hercules superclusters (at R.A. 12^h and redshift 7000 km/s, and 15^h and redshift 12000 km/s, respectively); on right panel the Perseus Supercluster at R.A. 1^h and redshift 5000 km/s, and the northern part of the Hercules supercluster. Also filaments of galaxies joining superclusters, and voids between superclusters are well seen. Long-dashed lines indicate the zone of avoidance.

Already in 1975, after the Tbilisi Meeting, we discussed with Zeldovich the possibility to organise a real international conference devoted solely to cosmology. Due to Soviet bureaucratic system it was extremely difficult for Soviet

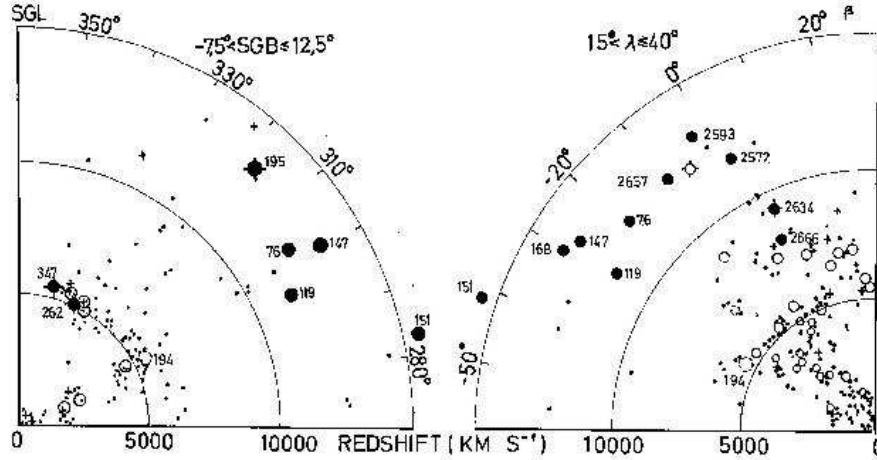


Figure 9. Wedge diagrams in supergalactic coordinates (left) and in coordinates parallel to the main chain of the Perseus supercluster (right) (Jõeveer and Einasto 1978). Designations as in Figure 8; Abell cluster names are indicated. The presence of galaxy filaments joining the Perseus and Local superclusters is well seen, as well as voids in front of and behind the Perseus supercluster. The last void has a depth from 5000 to 13000 km/s in redshift.

astronomers to attend international conferences in Western countries; thus the only possibility to have a better contact between Soviet and western cosmologists was to hold the conference within the Soviet Union. Zeldovich suggested to hold the symposium in Tallinn. After some discussion we decided to devote it to “Large Scale Structure of the Universe”. The symposium was held in September 1977. The chairman of the organizing committee was Malcolm Longair. He has spent a long period in Moscow and was well familiar with the work of Moscow theorists, and Zeldovich fully trusted him.

Our main results reported during the symposium by Jõeveer & Einasto (1978) were: (1) galaxies, groups and clusters of galaxies are not randomly distributed but form chains, concentrated in superclusters; (2) the space between galaxy chains contains almost no galaxies and form holes (voids) of diameter up to $\approx 70 h^{-1}$ Mpc (see Figure 9); (3) the whole picture of the distribution of galaxies and clusters resembles cells of a honeycomb, rather close to the picture predicted by Zeldovich.

The presence of holes (voids) in the distribution of galaxies were reported also by other groups: by Tully & Fisher (1978), Tift & Gregory (1978), and Tarenghi et al. (1978) in the Local, Coma and Hercules superclusters, respectively. A few years later a large void in Bootes was discovered by Kirshner et al. (1981). Theoretical interpretation of the observed cellular structure was discussed by Zeldovich (1978). As noted by Longair (1978) in his concluding remarks, *the discovery of the filamentary character of the distribution of galaxies,*

similar to a lace-tablecloth, and the overall cellular picture of the large-scale distribution was the most exciting result presented at this symposium. These results demonstrated that the pancake scenario by Zeldovich (1970) has many advantages over other rivalling scenarios. The term “Large-Scale Structure of the Universe” got its present meaning.



Figure 10. Jim Peebles explaining Scott Tremaine secrets of structure formation, IAU Tallinn Symposium 1977

A more detailed version of our results was published by Jõeveer, Einasto & Tago (1978). The next problem to solve was to find some explanation for the absence of galaxies in voids. This was done by Einasto, Jõeveer & Saar (1980). Saar developed an approximate model of the evolution of density perturbations in under- and over-dense regions based on Zeldovich (1970) ideas. He found that the matter flows out of under-dense regions and collects in over-dense regions until it collapses (pancake forming) and forms galaxies and clusters. In under-dense regions the density decreases continuously, but never reaches zero: there must be primordial matter in voids. Galaxy formation occurs not everywhere but only after the matter has collapsed to pancakes. Originally we believed that pancakes are 2-dimensional surfaces as predicted by Zeldovich (1970). To our surprise we did not find evidence for the presence of wall-like structures between voids – *the dominating structural element was a chain (filament) of galaxies and clusters*. Later the absence of wall-like pancakes and the dominance of filaments was explained theoretically by Bond, Kofman & Pogosyan (1996).

Our first 3-dimensional picture of the distribution of galaxies was based on the Second Reference Catalogue of Galaxies by de Vaucouleurs, de Vaucouleurs, & Corwin (1976). As new redshifts became available (Sandage & Tammann 1981, the first Harvard Center for Astrophysics redshift survey, and ZCAT by Huchra 1981), we made more detailed analyses of the Local, Coma, Perseus

and Hercules superclusters, and the huge void between these superclusters. In collaboration with Dick Miller we prepared several movies where the third dimension was visualised by rotation of the galaxy or cluster sample (Einasto & Miller 1983). Also quantitative methods were applied to describe the structure – correlation function, cluster and percolation analysis, multiplicity analysis of galaxy systems (Einasto, Klypin & Shandarin 1983, Einasto et al. 1984). Our results were summarised in a review article by Zeldovich, Einasto & Shandarin (1982) with the following principal conclusions. Most galaxies and clusters are concentrated in superclusters and are aligned along strings; strings join in central rich clusters of galaxies; the whole distribution is cellular. Large-scale structure of the Universe changes slowly, thus the present structure reflects the history of its formation and evolution. The space between galaxy and cluster chains and superclusters is almost devoid of visible objects; galaxies form only in dense and cold gas, thus no galaxies will be formed in interiors of voids and intensive galaxy formation occurs along lines (chains) and knots (clusters). Cluster and multiplicity analysis showed that there were several important differences between models and observations: in all models there was a population of rarefied particles in voids not represented as galaxies in the real Universe (compare Figures 7, 8 and 9). The connectivity of systems was in agreement with observations only in the pancake model (in contrast to the hierarchical clustering model). No model yielded a multiplicity function in agreement with observations: in the real Universe systems of galaxies of all multiplicity exist, in hierarchical models there are no large supercluster-type systems, pancake models yield no systems of intermediate and small multiplicity (there is no fine structure of small filaments in voids as in the real Universe).

4.3. Cold Dark Matter, Biasing and Regularity of the Structure

In late 1981 I was visiting ESO to analyse the large scale structure and to prepare with Dick Miller movies on the distribution of galaxies. At this time Oort was preparing a review paper on superclusters (Oort 1983). To discuss the structure of superclusters with Jan Oort, I had to make a special arrangement. Under Soviet rules I had no permission to visit the Netherlands during this visit, so we agreed with Oort to meet in Bonn, the German observatory closest to the Netherlands. After the discussion we agreed that there exists strong evidence for the formation of galaxies in chains: the velocity dispersion of groups and galaxies perpendicular to the chain axis is practically zero; main galaxies in clusters are elongated along the axis of the chain (as seen in the Perseus chain by Jõeveer, Einasto & Tago 1978 and Einasto, Jõeveer & Saar 1980). These results were important in several aspects. First, it was the first demonstration of biased galaxy formation (no galaxies form in voids). Secondly, galaxy formation occurs *in situ* in chains and clusters. Third, all previous structure formation scenarios had weak points, and a new better scenario had to be suggested. A new scenario was suggested, among others, by Bond, Szalay & Turner (1982); here hypothetical particles like axions, gravitinos or photinos play the role of dark matter. Numerical simulations of structure evolution for neutrino and axion-gravitino-photino-dominated universe were made and analysed by Melott et al. (1983). All quantitative characteristics (connectivity of the structure, multiplicity of galaxy systems, correlation function) of this new model fit the

observational data well. This model was called subsequently the Cold Dark Matter (CDM) model, in contrast to the neutrino-based Hot Dark Matter model. Presently the CDM model with some modifications is the most accepted model of the structure evolution (Blumenthal et al. 1984).

The presence of voids and superclusters was generally accepted after the work of the Harvard group (Geller & Huchra 1989). It is a bit strange that Harvard astronomers initially avoided the term “void” and used a new term, “bubble”; also they coined the terms “walls” and “great walls” for filaments and superclusters, respectively.

In the 1980s our cosmology group was expanded by 4 young cosmologists Lev Kofman, Dmitri Pogosyan, Maret Einasto and Mirt Gramann. Kofman studied the theory of inflation (with Andrei Linde) and models dominated by cosmological term (together with Alexei Starobinsky). Following a suggestion by Enn Saar, Mirt Gramann performed numerical simulations of models with the cosmological term, initially (in 1984) with a small number of particles. Soon Tartu Observatory got its first UNIX computer with 2 MB of core memory; so it was possible to simulate 3-D models with 64^3 particles and mesh. One run took about a month – not too much at those days. The main arguments in favour of this model were: direct density estimates of the mean matter density (including dark matter in voids and galaxy systems) yield a low value of the density parameter, $\Omega_0 \approx 0.2$; only a model with the cosmological term fits simultaneously data on the Hubble constant and the age of the Universe. Inflation models predict that the total density of the Universe is equal to the critical density which is possible only in the presence of a cosmological constant (Starobinsky 1982, Kofman & Starobinsky 1985). This model was calculated by Gramann (1988) and was compared with observational data using various quantitative methods (Einasto, Einasto & Gramann 1989, Gramann 1990, Einasto et al. 1991). All tests suggested that this model fits all observational data very well. This LCDM model was probably the first use of the presently popular model with the cosmological term. Independent observational evidence favouring a CDM model with the cosmological term was found by Efstathiou et al. (1990).

One major issue was the study of physical biasing. Here we found that the most important effect is the absence of galaxy formation in voids and the presence of primordial dark matter here (Einasto, Jõeveer & Saar 1980, Einasto & Saar 1987). The presence of an almost homogeneous population in voids increases the amplitude of the power spectrum of galaxies in respect to matter; the amplitude shift is determined by the fraction of matter in the clustered population associated with galaxies (Einasto et al. 1994a, 1999b).

Kofman, Einasto and Linde (1987) discussed voids (cosmic bubbles) as remnants from inflation. Kofman and Shandarin (1988) invented the adhesion model of the evolution of the large scale structure. This model uses simple geometric constructions to calculate, in the first approximation, the formation and further evolution of the filamentary web of cosmic structures using the initial density perturbation field.

In the 1990s the main attention of our group was directed to the detailed analysis of the regularity of the supercluster–void network. Some hints to the concentration of clusters of galaxies to a dominant plane close to the supergalactic plane were known already in the 1980s (Einasto & Miller 1983, Tully et al.

1992). This issue became topical after the discovery by Broadhurst et al. (1990) that the distribution of high-density regions may be quasi-regular or periodic. Broadhurst et al. found such a regularity in the direction of the north and south Galactic poles. Our study confirmed that this regularity is indeed 3-dimensional – high-density regions marked by rich superclusters form a quasi-regular lattice (Einasto et al. 1994b, 1997a,b,c,d). This regularity may be caused by the presence of a peak or bump in the primordial power spectrum of matter (Einasto et al. 1999a,b,c). The presence of such feature is predicted in some variants of the inflation theory (Lesgourgues, Polarski, & Starobinsky, 1998, Chung et al. 1999). The issue is still not solved; it is possible that a paradigm shift is needed here, since no generally accepted theory of structure formation yields a regular structure.

5. Epilogue

In the spring of 1973 there was a conference on galaxies in Tbilisi, and one evening I walked with Rashid Sunyaev along the Rustaveli avenue discussing our galaxy models with dark halos. Sunyaev argued that nobody will take our results seriously until some American astronomer confirmed them. Indeed, our experience has confirmed Sunyaev prediction several times. Öpik's measurement of the distance of the Andromeda galaxy was accepted only after Hubble's (1925) discovery of Cepheids in M31 and other galaxies of the Local Group. Kuzmin's determination of the local mass density in the Universe remained almost unknown, even after the work by Gilmore, Wyse and Kuijken. Our work on the presence of dark matter around galaxies was noticed after Ostriker, Peebles and Yahil (1974) got similar results, and Vera Rubin confirmed flat rotation curves of galaxies; the void-filament structure of the universe was accepted after the work by Geller and Huchra (1989). Is this a rule in scientific revolutions?

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